STATE ESTIMATION

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1 Experiment description

Our robots seem to not know how fast they are exactly going. Reported speeds have a constant relative error of 1.12 (in the forward direction) and 1.1 (in the sideways direction). In order to determine where this issue is located this report will check if the wheel speeds are accurately being measured by the robot.
This has been done by placing a robot on top of a roll of tape. A tachometer will be used to measure the actual speeds of the wheels. For consistent measurements to happen, the robot will not be using any form of control, allowing for consistent wheel speeds.
The robot will be given a step response of either 1 or 2 m/s. The speeds of the wheels will be measured multiple times, during multiple runs.

2 Results

In Table 1 one can find the relative difference between the speed that is reported (R) by the robot and the measured (M) speed by the tachometer for each wheel of the robot. The raw data that has been used for this table can be found in Appendix A.

Wheel	R / M (1 m/s)	R / M (2 m/s)
RF	0.971	0.988
LF	1.017	1.00
LB	1.030	1.00
RB	0.957	0.976

Table 1: Resulting difference between the reported and measured wheel speeds

When using these factors to calculate our expected speed when would want to go 1 m/s in the pseudo inverse of our velocity coupling matrix $D^\dagger$ (Equation 1) we get the result as shown in Equation 2. As one can see, the forward velocity only differs by 0.005 m/s. Which is nowhere close to a difference of 1.12.

$$D^\dagger(\theta,\phi) = \frac{1}{2}\begin{pmatrix} -\frac{1}{\sin\theta+\sin\phi} & -\frac{1}{\sin\theta+\sin\phi} & \frac{1}{\sin\theta+\sin\phi} & \frac{1}{\sin\theta+\sin\phi} \\ \frac{\cos\phi}{\cos^2\theta+\cos^2\phi} & -\frac{\cos\phi}{\cos^2\theta+\cos^2\phi} & -\frac{\cos\theta}{\cos^2\theta+\cos^2\phi} & \frac{\cos\theta}{\cos^2\theta+\cos^2\phi} \\ \frac{\sin\theta}{\sin\theta+\sin\phi} & \frac{\sin\theta}{\sin\theta+\sin\phi} & \frac{\sin\phi}{\sin\theta+\sin\phi} & \frac{\sin\phi}{\sin\theta+\sin\phi} \end{pmatrix} \tag{1}$$

$$
\begin{pmatrix}
-\frac{1}{\sin 60^\circ + \sin 30^\circ} & -\frac{1}{\sin 60^\circ + \sin 30^\circ} & \frac{1}{\sin 60^\circ + \sin 30^\circ} & \frac{1}{\sin 60^\circ + \sin 30^\circ} \\
\frac{\cos 30^\circ}{\cos^2 60^\circ + \cos^2 30^\circ} & -\frac{\cos 30^\circ}{\cos^2 60^\circ + \cos^2 30^\circ} & -\frac{\cos 60^\circ}{\cos^2 60^\circ + \cos^2 30^\circ} & \frac{\cos 60^\circ}{\cos^2 60^\circ + \cos^2 30^\circ} \\
\frac{\sin 60^\circ}{\sin 60^\circ + \sin 30^\circ} & \frac{\sin 60^\circ}{\sin 60^\circ + \sin 30^\circ} & \frac{\sin 30^\circ}{\sin 60^\circ + \sin 30^\circ} & \frac{\sin 30^\circ}{\sin 60^\circ + \sin 30^\circ}
\end{pmatrix}
\times \begin{pmatrix} 0.866 \times 0.980 \\ -0.866 \times 1.01 \\ -0.5 \times 1.02 \\ 0.5 \times 0.967 \end{pmatrix} = \begin{pmatrix} 0.000 \\ 0.995 \\ -0.162 \end{pmatrix} = \begin{pmatrix} v_u \\ v_v \\ v_\omega \end{pmatrix} \quad (2)
$$

3 Conclusions

After this more experiments followed where RPMs have been directly logged from the robot and have been compared with RPMs as measured with the help of the tachometer. These all show that the robot knows its wheel speeds accurately.
One interesting takeaway is that the left wheels seemed to perform more accurately that the right wheels on this specific robot. Since no other robots have unfortunately been available for testing we cannot tell if this robot specific or a consistent issue.
Our next hypothesis is that perhaps the angles of the wheels are slightly off. Causing a consistent error in the state estimation.

4 Follow-up check

As discussed during the conclusions we assume that perhaps the angles are slightly off on the actual robot. That is why we will calculate the needed angles that cause us to achieve an offset of 1.12 and 1.1. Given the instructed wheel speeds that one would have with wheels at 30 and 60 degrees.

$$
\begin{pmatrix} -\sin\phi & \cos\phi & 1 \\ -\sin\phi & -\cos\phi & 1 \\ \sin\theta & -\cos\theta & 1 \\ \sin\theta & \cos\theta & 1 \end{pmatrix} \times \begin{pmatrix} 1.1 \\ 1.12 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}-1}{2} \\ -\left(\frac{\sqrt{3}+1}{2}\right) \\ \frac{\sqrt{3}-1}{2} \\ \frac{\sqrt{3}+1}{2} \end{pmatrix} \quad (3)
$$

It seems that there is not a proper answer to this question. When one assumes that ϕ and θ are identical (as one should) the answers for this system are ϕ = 39.4° and θ = 63.5°. However, when you solve the equations separately then ϕ can either be 32°, 15° or 74°. And θ can be either 59°, 15° or 74°.

A Raw data

The RPM is measured with the help of a tachometer (testo 470). The linear velocity is reported by the robot. During all of these tests both the Kalman filter and the x, y and wheel PID controllers were disabled. This allows for more consistent wheel velocities, allowing us to take average measurements of longer intervals, increasing precision.
Every test consists out of a step response that is given to the robot.

A.1 1 m/s

All the tests where conducted by commanding the robot to go 1 m/s.

Test	Wheel	RPM	Linear velocity
120049	RF	263.1	0.746
120741	RF	262.6	0.747
120854	RF	262.0	0.747
121514	RF	261.9	0.747
121726	RF	261.8	0.748
120149	LF	250.6	-0.755
120642	LF	254.2	-0.758
120951	LF	254.7	-0.758
121413	LF	254.1	-0.754
121840	LF	253.4	-0.754
120308	LB	130.3	-0.401
120556	LB	126.2	-0.381
121045	LB	126.1	-0.381
121325	LB	127.4	-0.380
121930	LB	126.7	-0.380
120405	RB	123.6	0.351
120507	RB	124.3	0.351
121137	RB	123.4	0.339
121243	RB	124.0	0.350
122022	RB	125.5	0.351

Table 2: Raw data for 1 m/s

A.2 2 m/s

All the test where conducted by commanding the robot to go 2 m/s.

Test	Wheel	RPM	Linear velocity
151900	RF	575.1	1.664
152113	RF	573.4	1.660
152204	RF	572.0	1.659
152258	RF	572.5	1.660
152346	RF	572.5	1.659
152452	LF	564.5	-1.670
152542	LF	567.3	-1.661
152629	LF	567.6	-1.671
152716	LF	562.5	-1.665
152850	LF	567.1	-1.661
152950	LB	302.6	-0.899
153038	LB	290.9	-0.860
153127	LB	301.8	-0.905
153234	LB	307.4	-0.911
153350	LB	306.3	0.871
153449	RB	308.7	0.886
153545	RB	308.4	0.891
153638	RB	305.6	0.873
153733	RB	305.6	0.869
153828	RB	306.1	0.870

Table 3: Raw data for 2 m/s

B Calculation of expected wheelspeeds

In order to go 1 m/s in a pure forward direction the wheels will have to achieve the following speeds:

$$D(\theta, \phi) = \begin{pmatrix} -\sin\phi & \cos\phi & 1 \\ -\sin\phi & -\cos\phi & 1 \\ \sin\theta & -\cos\theta & 1 \\ \sin\theta & \cos\theta & 1 \end{pmatrix} \tag{4}$$

$$D\left(60°, 30°\right) = \begin{pmatrix} -\sin 30° & \cos 30° & 1 \\ -\sin 30° & -\cos 30° & 1 \\ \sin 60° & -\cos 60° & 1 \\ \sin 60° & \cos 60° & 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 1 \end{pmatrix} \tag{5}$$

The speed that we will have to achieve for each wheel is:

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \approx \begin{pmatrix} 0.866 \\ -0.866 \\ -0.5 \\ 0.5 \end{pmatrix} \text{m/s} \tag{6}$$

C Follow-up check equations

This section will elaborate on how this equation has been solved:

$$\begin{pmatrix} -\sin\phi & \cos\phi & 1 \\ -\sin\phi & -\cos\phi & 1 \\ \sin\theta & -\cos\theta & 1 \\ \sin\theta & \cos\theta & 1 \end{pmatrix} \times \begin{pmatrix} 1.1 \\ 1.12 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}-1}{2} \\ -\left(\frac{\sqrt{3}+1}{2}\right) \\ \frac{\sqrt{3}-1}{2} \\ \frac{\sqrt{3}+1}{2} \end{pmatrix} \tag{7}$$

At first we will rewrite this system into separate formulas

$$-1.1\sin\phi + 1.12\cos\phi = \frac{\sqrt{3}-1}{2} \tag{8}$$

$$-1.1\sin\phi - 1.12\cos\phi = -\left(\frac{\sqrt{3}+1}{2}\right) \tag{9}$$

$$1.1\sin\theta - 1.12\cos\theta = \frac{\sqrt{3}-1}{2} \tag{10}$$

$$1.1\sin\theta + 1.12\cos\theta = \frac{\sqrt{3}+1}{2} \tag{11}$$

Remark: A very confusing result occurs. When one asks a math solver to find a solution for any of these equations one gets the following answers. For Equation 8 we get $\phi = 32°$; for Equation 9 and 11 we get $\phi = \theta = 15° \vee 74°$; and for Equation 10 we get $\theta = 59°$. These solutions suggest that there can not really be a proper solution since the angles are not consistent. For completeness below we assume that $\phi = \phi$ and $\theta = \theta$.

Then we rewrite Equations 8 and 10 to have the $\sin$ on the left-hand side.

$$1.1\sin\phi = -\left(\frac{\sqrt{3}-1}{2}\right) + 1.12\cos\theta \tag{12}$$

$$\sin\phi = -\left(\frac{\sqrt{3}-1}{2.2}\right) + \frac{1.12}{1.1}\cos\theta \tag{13}$$

$$1.1\sin\theta = \frac{\sqrt{3}-1}{2} + 1.12\cos\theta \tag{14}$$

$$\sin\theta = \frac{\sqrt{3}-1}{2.2} + \frac{1.12}{1.1}\cos\theta \tag{15}$$

Next, we substitute the resulting Equation 13 into Equation 9. Similar we substitute Equation 15 into Equation 11.

$$-1.1\left[-\left(\frac{\sqrt{3}-1}{2.2}\right) + \frac{1.12}{1.1}\cos\phi\right] - 1.12\cos\phi = -\left(\frac{\sqrt{3}+1}{2}\right) \tag{16}$$

$$1.1\left[\frac{\sqrt{3}-1}{2.2} + \frac{1.12}{1.1}\cos\theta\right] + 1.12\cos\theta = \frac{\sqrt{3}+1}{2} \tag{17}$$

Finally, Equation 16 can be solved for $\cos\phi$.

$$\frac{\sqrt{3}-1}{2} - 1.12\cos\phi - 1.12\cos\phi = -\left(\frac{\sqrt{3}+1}{2}\right) \tag{18}$$

$$\frac{\sqrt{3}-1+\sqrt{3}+1}{2} = 2.24\cos\phi \tag{19}$$

$$\sqrt{3} = 2.24\cos\phi \tag{20}$$

$$\cos\phi = \frac{\sqrt{3}}{2.24} \tag{21}$$

$$\phi = \cos^{-1}\frac{\sqrt{3}}{2.24} \approx 39.4° \tag{22}$$

Similarly Equation 17 can be solved for $\cos\theta$.

$$\frac{\sqrt{3}-1}{2} + 1.12\cos\theta + 1.12\cos\theta = \frac{\sqrt{3}+1}{2} \tag{23}$$

$$\frac{\sqrt{3}-1-\sqrt{3}-1}{2} = -2.24\cos\theta \tag{24}$$

$$-1 = -2.24\cos\theta \tag{25}$$

$$\cos\theta = \frac{1}{2.24} \tag{26}$$

$$\theta = \cos^{-1}\frac{1}{2.24} \approx 63.5° \tag{27}$$

Hence resulting in ϕ being 39.4°and θ being 63.5°.